Pearson Edexcel

Mark Scheme (Results)

January 2020

Pearson Edexcel International Advanced Level In Core Mathematics C34 (WMA02) Paper 01

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PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 125

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - ∗ The answer is printed on the paper or ag- answer given
   - ⎣ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \ldots$

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to $x = \ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.
Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

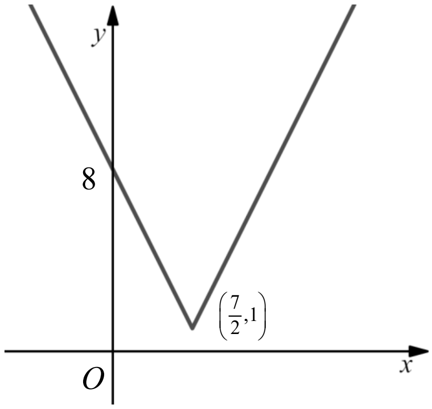
Question Number	Scheme		Marks
1(a)	$\left(f'(x)=\right)8x^3+2x-3=0$	Attempts to differentiate (reduction of power by 1 seen at least once including 8→ 0) and sets their $f'(x)=0$ which may be implied by subsequent work.	M1
	$x^3=\frac{3-2x}{8}$ or $8x^3=3-2x$	Makes x^3 or kx^3 the subject of their $f'(x)=0$ where the x^3 or kx^3 has come from differentiating $2x^4$. **Dependent on the previous M.**	**d**M1
	$x=\sqrt[3]{\frac{3-2x}{8}}$ * or $x=\sqrt[3]{\frac{-2x+3}{8}}$ *	Obtains the printed answer with no errors or omissions. Allow use of α rather than x for the M marks but the final answer must be in terms of x. Be generous if the cube root does not fully encompass the expression.	A1*
			(3)
	Alternative for **d**M1 A1 in (a):		
	$x=\sqrt[3]{\frac{3-2x}{8}}\Rightarrow x^3=\frac{3-2x}{8}\Rightarrow 8x^3+2x-3=0$ Cubes both sides of the given equation and rearranges to obtain g(x) = 0		**d**M1
	Which is $f'(x)=0$ therefore proved	Obtains $8x^3+2x-3=0$ both times (including the "= 0") and makes a (minimal) conclusion.	A1
(b)	$x_2=\sqrt[3]{\frac{3-2\times 0.6}{8}}=...$	Substitutes x_1 = 0.6 into the given formula to find a value for x_2. This may be implied by their expression or awrt 0.61	M1
	$\Rightarrow x_2$ = awrt 0.6082, x_3 = awrt 0.6064 Both values correct which round to the above. Mark in order that the values appear and ignore how they are referenced and ignore any further iterations.		A1
	Note that some candidates take the square root rather than the cube root and this scores M0 in (b) if there is no evidence that they have used the correct formula. (Values to look for are 0.4743… and 0.5063…)		
			(2)
(c)	$f'(0.6065)=-0.0022...$ $f'(0.6075)=0.0086...$	Chooses a suitable interval for x, which is within 0.607 ± 0.0005 and attempts to evaluate their $f'(x)$ for both values (must be a 'changed' f(x)) although they might refer to it as f(x).	M1
	Note that it is possible to use $g(x)=x-\sqrt[3]{\frac{3-2x}{8}}$ which gives g(0.6065) = –0.0002524… and g(0.6075) = 0.00097401 but if it is not clear which function is being used then score M0. Note that many candidates use f (x) giving values of 6.8189580… and 6.8189612… and also scores M0		
	Sign change (negative, positive) therefore root. Both values correct awrt (or truncated) 1 sf, sign change (or e.g. < 0, > 0 or f′(0.6065).f′(0.6075) < 0 or f′(0.6065) < 0 < f′(0.6075)) and a minimal conclusion e.g. therefore root. Allow tick, QED, hash, square box, smiley face etc.		A1
	Attempts at successive iteration score M0 in (c)		
			(2)
			[7 marks]

Question Number	Scheme		Marks
2(a)	$\left(\frac{1}{4}-3x\right)^{\frac{1}{2}}=\frac{1}{2}(1\pm...)^{\frac{1}{2}}$	Takes out a common factor of $\sqrt{\left(\frac{1}{4}\right)}$ or $\frac{1}{2}$ or equivalent e.g. $\frac{1}{\sqrt{4}}, \left(\frac{1}{4}\right)^{\frac{1}{2}}, 2^{-1}, 4^{-\frac{1}{2}}$ to give $\frac{1}{2}(1\pm...)^{\frac{1}{2}}$ oe	B1
	$(1-12x)^{\frac{1}{2}}=1-\left(\frac{1}{2}\right)12x+\frac{\left(\frac{1}{2}\right)\times\left(\frac{1}{2}-1\right)}{2!}\times(-12x)^2+\frac{\left(\frac{1}{2}\right)\times\left(\frac{1}{2}-1\right)\times\left(\frac{1}{2}-2\right)}{3!}\times(-12x)^3$ For the binomial expansion of $(1+ax)^{\frac{1}{2}}$ where $a\neq -3$ Award for a correct structure for term three and/or term 4 (allow ±'12'x) Condone the omission of brackets. E.g. allow $\frac{\frac{1}{2}\times\frac{1}{2}-1\times\frac{1}{2}-2}{3!}\times"12"x^3$ for term 4		M1
	$(1-12x)^{\frac{1}{2}}=1-\left(\frac{1}{2}\right)12x+\frac{\left(\frac{1}{2}\right)\times\left(\frac{1}{2}-1\right)}{2!}\times(-12x)^2+\frac{\left(\frac{1}{2}\right)\times\left(\frac{1}{2}-1\right)\times\left(\frac{1}{2}-2\right)}{3!}\times(-12x)^3$ or $(1-12x)^{\frac{1}{2}}=1-6x-18x^2-108x^3-...$ This mark is for a correct unsimplified or simplified expansion of $(1-12x)^{\frac{1}{2}}$ If unsimplified, the brackets must be present where necessary unless they are implied by subsequent work. Allow $(12x)^2$ for term 3.		A1
	$=\frac{1}{2}-3x-9x^2-54x^3+...$	Any 2 correct simplified terms	A1
		All correct and simplified	A1
	Special case: **If all the working is correct but the brackets are not removed e.g.** $\frac{1}{2}\left(1-6x-18x^2-108x^3-...\right)$ **Score B1M1A1A1A0**		
			(5)
(a) Way 2 (Direct Expansion)	$\left(\frac{1}{4}-3x\right)^{\frac{1}{2}}=\left(\frac{1}{4}\right)^{\frac{1}{2}}+\frac{1}{2}\left(\frac{1}{4}\right)^{-\frac{1}{2}}(-3x)+\frac{\left(\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)}{2!}\left(\frac{1}{4}\right)^{-\frac{3}{2}}(-3x)^2+\frac{\left(\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{3!}\left(\frac{1}{4}\right)^{-\frac{5}{2}}(-3x)^3+...$ B1: For first term $\left(\frac{1}{4}\right)^{\frac{1}{2}}$ or as defined above M1: For a correct structure for term three and/or term 4. (allow ±3x) A1: Correct and unsimplified binomial expansion. The brackets must be present where necessary unless they are implied by subsequent work.		B1 M1 A1
	$=\frac{1}{2}-3x-9x^2-54x^3+...$	Any 2 correct simplified terms	A1
		All correct and simplified	A1
(b)	$\sqrt{22}\approx 10\left(\frac{1}{2}-\frac{3}{100}-\frac{9}{10000}-\frac{54}{1000000}\right)$ Substitutes $x=\frac{1}{100}$ into their expansion and multiplies by 10 to obtain a value. You may need to check if no working is shown.		M1
	$\left(\sqrt{22}=\right)4.6905$	Correct value only	A1
			(2)
			[7 marks]

Question Number	Scheme		Marks
3(a)			

x	4	4.5	5	5.5	6
y	$\frac{10}{1+\sqrt{4}}$	$\frac{10}{1+\sqrt{4.5}}$	$\frac{10}{1+\sqrt{5}}$	$\frac{10}{1+\sqrt{5.5}}$	$\frac{10}{1+\sqrt{6}}$
y	$\frac{10}{3}$	$\frac{-20+30\sqrt{2}}{7}$	$\frac{-5+5\sqrt{5}}{2}$	$\frac{-20+10\sqrt{22}}{9}$	$-2+2\sqrt{6}$
y	3.33333…	3.20377…	3.09016…	2.98935…	2.89897…

Question Number	Scheme		Marks
3(a)	Attempts at least 3 values for y as shown. Either in exact or decimal form. Must be accurate to 2dp for decimals unless implied by the correct answer later		M1
	$h = 0.5$	Correct strip width. May be implied by their x values.	B1
	Area $\approx \frac{0.5}{2}\{3.333+2.899+2\times(3.204+3.090+2.989)\} = ...$ Fully correct application of the trapezium rule e.g. $\frac{h}{2}\{\text{Correct } y \text{ value structure}\}$ Allow a correct y value structure for their y values but must be for at least 3 x values that include y values at $x = 4$ and $x = 6$ E.g. $\approx \frac{1}{2}(1)\{3.333+2.899+2\times(3.090)\} = ...$ scores M1B0M1A0		M1
	$= 6.20$	Allow awrt 6.20 but also allow 6.2 **but not** awrt 6.2	A1
			(4)
(b)	• **In (b) the method must be made clear as required by the question** • **Correct or correct ft answers with no working score no marks** • **Attempts to use the trapezium rule again score no marks** • **NB integration/calculator gives (i) 18.5925… (ii) 12.1975…**		
(i)	$"6.20"\times 6 = ...$ or $"6.20"\div 2 = ...$ or $"6.20"\times 3 = ...$	Allow for any one of: • Answer to (a) × 6 **only** • Answer to (a) ÷ 2 **only** • Answer to (a) × 3 **only** **(Not necessarily evaluated)**	M1
	18.60	Answer to (a) × 3. If correct, allow awrt 18.6. For ft be generous and allow answers that are clearly (a) × 3	A1ft
(ii)	$\int_4^6 \frac{13+3\sqrt{x}}{1+\sqrt{x}}\,dx = "6.20" + 6 = ...$	Their (a) value + 6. Note that it is acceptable for the "6" to come from $\int_4^6 3\,dx$	M1
	12.20	Answer to (a) + 6. If correct, allow awrt 12.2. For ft be generous and allow answers that are clearly (a) + 6	A1ft
			(4)
			[8 marks]

Question Number	Scheme		Marks
4(a)		A V-shape anywhere. (Ignore gradient as long as it is a V shape) Do not be overly concerned by lack of symmetry and ignore any extra dashed or dotted lines.	B1
		A V-shape with intercept at (0, 8) or 8 marked on the y-axis or (8, 0) marked in the correct place on the y-axis. Their graph must cross (not just touch) the y-axis to score this mark Allow away from the sketch but must be (0, 8). The sketch has precedence.	B1
		Min point at $\left(\frac{7}{2},1\right)$ which must correspond with the sketch i.e. in quadrant 1. Ignore any other minimum points.	B1
			(3)
(b) Way 1	$14-x=2x-7+1 \Rightarrow x=...$ **or** $14-x=-2x+7+1 \Rightarrow x=...$	Attempts to solve one of these equations or equivalent	M1
	Either $x=\frac{20}{3}$ **or** $x=-6$	One correct value	A1
	$14-x=2x-7+1 \Rightarrow x=...$ **and** $14-x=-2x+7+1 \Rightarrow x=...$	Attempts to solve both of these equations or equivalents. **Dependent on the previous M.**	**d**M1
	$x=\frac{20}{3}$ **and** $x=-6$	Both correct values and no other x values. Ignore any attempts at y values.	A1
			(4)
(b) Way 2	$14-x=\lvert 2x-7\rvert+1 \Rightarrow \lvert 2x-7\rvert=13-x$ $\Rightarrow (2x-7)^2=(13-x)^2$	Isolates $\lvert 2x-7\rvert$ and attempts to square both sides	M1
	$\Rightarrow 3x^2-2x-120=0$	Correct quadratic equation	A1
	$\Rightarrow (3x-20)(x+6)=0$ $x=...$	Solves their 3TQ (usual rules). **Dependent on the previous M.**	**d**M1
	$x=\frac{20}{3}$ **and** $x=-6$	Both correct values and no other x values. Ignore any attempts at y values.	A1
(c)	$"1"=\frac{1}{2}\times"\frac{7}{2}"+k \Rightarrow k=...$	Uses $y=\frac{1}{2}x+k$ with their (3.5, 1) where their $3.5\neq 0$ to find 'k'	M1
	$k<-\frac{3}{4}$	Allow equivalent notation e.g. $(-\infty, -0.75)$	A1
			(2)
			[9 marks]

Question Number	Scheme		Marks
5(a)	$f(x) \leqslant 27$	Allow $y \leqslant 27$, range $\leqslant 27$, $(-\infty, 27]$, $f \leqslant 27$ but not $x \leqslant 27$	B1
			(1)
	Mark (i) and (ii) together		
(b)(i)	$9+3x=0 \Rightarrow x=-3$	$x=-3$. Allow $x=-\frac{9}{3}$	B1
(ii)	$f(12)=0 \Rightarrow B-144A=0$ **or** $f(6)=27 \Rightarrow B-36A=27$	Uses $x = 12$ and $y = 0$ **or** $x = 6$ and $y = 27$ in $y = B - Ax^2$ (i.e. uses $x = 6$ in $B - Ax^2 = 9 + 3x$)	M1
	$f(12)=0 \Rightarrow B-144A=0$ **and** $f(6)=27 \Rightarrow B-36A=27$ $\Rightarrow A=\ldots, B=\ldots$	Uses $x = 12$ and $y = 0$ **and** $x = 6$ and $y = 27$ in $y = B - Ax^2$ (i.e. uses $x = 6$ in $B - Ax^2 = 9 + 3x$) and obtains values for A and B	M1
	$A=\frac{1}{4}, B=36$	Correct values	A1
			(4)
(c)	$ff(0)=f(9)=36-\frac{9^2}{4}=\frac{63}{4}$	Attempts $B \pm A \times 9^2$ with their values of A and B	M1
		15.75 oe (15.8 scores A0 unless 15.75 is seen earlier then isw)	A1
			(2)
			[7 marks]

Question Number	Scheme		Marks
6.	$\int y\,\mathrm{d}y = \int 4x\ln x\,\mathrm{d}x$ or e.g. $\int \frac{y}{4}\,\mathrm{d}y = \int x\ln x\,\mathrm{d}x$ Separates the variables. Allow without the integral signs but must include the dx and dy unless they are implied by subsequent work.		B1
	$\int kx\ln x\,\mathrm{d}x \to Ax^2\ln x - \int \frac{Bx^2}{x}\,\mathrm{d}x$ This mark is for applying integrating by parts to the RHS to obtain an expression of this form		M1
	$\int y\,\mathrm{d}y = \frac{y^2}{2}(+c)$ or $\int \frac{y}{4}\,\mathrm{d}y = \frac{y^2}{8}(+c)$	Integrates the LHS correctly with or without "+ c"	B1
	$\int 4x\ln x\,\mathrm{d}x = 2x^2\ln x - x^2\,(+c)$ or $\int x\ln x\,\mathrm{d}x = \frac{x^2}{2}\ln x - \frac{x^2}{4}(+c)$ Integrates the RHS correctly with or without "+ c"		A1
	$\frac{4^2}{2} = 2(1)^2\ln 1 - (1)^2 + c \Rightarrow c = ...$ or e.g. $\frac{4^2}{8} = \frac{1}{2}\ln 1 - \frac{(1)^2}{4} + c \Rightarrow c = ...$	Substitutes $x = 1$ and $y = 4$ into an equation formed from some integration in an attempt to find c	M1
	$x = \mathrm{e} \Rightarrow \frac{y^2}{2} = 2\mathrm{e}^2\ln \mathrm{e} - \mathrm{e}^2 + 9 \Rightarrow y^2 = ...$ or $y = ...$ $x = \mathrm{e} \Rightarrow \frac{y^2}{8} = \frac{\mathrm{e}^2}{2}\ln \mathrm{e} - \frac{\mathrm{e}^2}{4} + 2.25 \Rightarrow y^2 = ...$ or $y = ...$ **Dependent upon both M's**. Scored for a full method to find y or y^2 when $x = \mathrm{e}$		**dd**M1
	$y = \sqrt{2\mathrm{e}^2 + 18}$	Cao ($y = \pm\sqrt{2\mathrm{e}^2 + 18}$ is A0 and $y = \sqrt{4\mathrm{e}^2 - 2\mathrm{e}^2 + 18}$ is A0) **but apply isw if necessary.**	A1
			(7)
			[7 marks]

Question Number	Scheme		Marks
7(a)	$y = 3x(2x-5)^4 \Rightarrow \frac{dy}{dx} = 3(2x-5)^4 + 24x(2x-5)^3$ M1: $\frac{dy}{dx} = P(2x-5)^4 + Qx(2x-5)^3, \quad P,\ Q > 0$ If the product rule is quoted, it must be correct to score M1 A1: Correct differentiation (allow in any correct form)		M1 A1
	$\Rightarrow \frac{dy}{dx} = 3(2x-5)^3\{2x-5+8x\}$	Takes a common factor of $(2x-5)^3$ out of both terms. The factorisation must be correct for their expression and the powers of (2x -5) must be different.	M1
	$= 15(2x-5)^3(2x-1)$	Correct expression	A1
			(4)
(b)	$15(2x-5)^3(2x-1) = 0 \Rightarrow x = \frac{1}{2}, \frac{5}{2}$ Obtains **<u>both correct</u>** critical values – may be implied by e.g. $x < \frac{1}{2}\ x < \frac{5}{2}$ Also allow this mark for **one** correct 'end' e.g. $x < \frac{5}{2}$ or $x \leqslant \frac{5}{2}$ or $x > \frac{1}{2}$ or $x \geqslant \frac{1}{2}$		M1
	Examples: $\frac{1}{2} < x < \frac{5}{2}$ or $\frac{1}{2} \leqslant x \leqslant \frac{5}{2}$ $\frac{1}{2} < x \leqslant \frac{5}{2}$ or $\frac{1}{2} \leqslant x < \frac{5}{2}$ $\frac{1}{2} < x,\ x < \frac{5}{2}$ or $x \geqslant \frac{1}{2},\ x < \frac{5}{2}$ $\left(\frac{1}{2}, \frac{5}{2}\right)$ or $\left[\frac{1}{2}, \frac{5}{2}\right]$ $\left[\frac{1}{2}, \frac{5}{2}\right)$ or $\left(\frac{1}{2}, \frac{5}{2}\right]$	Acceptable region as shown	A1
			(2)
			[6 marks]

Question Number	Scheme		Marks
8 Way 1	$\int y\frac{dx}{dt}dt = \int 3\sin 2t \times 4\cos t\ (dt)$ Attempts $\int y\frac{dx}{dt}dt$ and obtains $k\int \sin 2t\cos t\ (dt)$		M1
	$= \int 3\times 2\sin t\cos t\times 4\cos t\ (dt)$ Uses the correct identity for $\sin 2t$ (may be implied) to obtain $\int A\sin t\cos^2 t\ (dt)$		M1
	$\int 24\sin t\cos^2 t\ (dt)$	Correct integral	A1
	$\int 24\sin t\cos^2 t\ (dt) = k\cos^3 t(+c)$ Correct form for the integration. Note that an equivalent form may be reached by substitution e.g. $u = \sin t$ gives $\int 24\sin t\cos^2 t\ dt = 24\int \frac{u(1-u^2)}{\sqrt{1-u^2}}\ du = 24\int u(1-u^2)^{\frac{1}{2}}\ du = -8(1-u^2)^{\frac{3}{2}}(+c)$ So in this case the mark can be awarded for obtaining $k(1-u^2)^{\frac{3}{2}}(+c)$ Or e.g. $u = \cos t$ gives $\int 24\sin t\cos^2 t\ dt = -24\int \frac{u^2\sqrt{1-u^2}}{\sqrt{1-u^2}}\ du = -24\int u^2\ du = -8u^3(+c)$ So in this case the mark can be awarded for obtaining $ku^3(+c)$		M1
	$= -8\cos^3 t$ Correct integration. Allow equivalent expressions e.g. $-8(1-u^2)^{\frac{3}{2}}, -8u^3$ as above.		A1
	$\left[-8\cos^3 t\right]_0^{\frac{\pi}{6}} = -8\left(\cos^3\frac{\pi}{6} - 8\cos^3 0\right)$ or e.g. $\left[-8(1-u^2)^{\frac{3}{2}}\right]_0^{\frac{1}{2}} = -8\left(\left(1-\frac{1}{2}\right)^{\frac{3}{2}} - (1-0)^{\frac{3}{2}}\right)$ or e.g. $\left[-8u^3\right]_1^{\frac{\sqrt{3}}{2}} = -8\left(\left(\frac{\sqrt{3}}{2}\right)^3 - (1)^3\right)$	Correct use of limits for their integrated function. Must see use of both limits. Allow use of 30° for $\frac{\pi}{6}$. **Dependent on the previous method mark.**	**d**M1
	$8-3\sqrt{3}$	Allow $8-\sqrt{27}$ **Depends on not having lost any of the previous marks.**	A1
			(7)

	Way 2: Factor Formulae		
	$\int y\frac{dx}{dt}dt = \int 3\sin 2t \times 4\cos t\ (dt)$ Attempts $\int y\frac{dx}{dt}(dt)$ and obtains $k\int \sin 2t\cos t\ (dt)$		M1
	$= 12\int \frac{1}{2}(\sin 3t + \sin t)\ (dt)$ Uses $\sin 2t\cos t = \frac{1}{2}(\sin 3t + \sin t)$ to obtain $\int A(\sin 3t + \sin t)(dt)$		M1
	$= 6\int (\sin 3t + \sin t)(\ dt)$	Correct integral	A1
	$6\int (\sin 3t + \sin t)\ (dt) = p\cos 3t + q\cos t(+c)$ Correct form for the integration.		M1
	$= -2\cos 3t - 6\cos t$	Correct integration	A1
	$\left[-2\cos 3t - 6\cos t\right]_0^{\frac{\pi}{6}}$ $-2\cos\pi - 6\cos\frac{\pi}{6} - (-2\cos 0 - 6\cos 0)$	Correct use of limits for their integrated function. Must see use of both limits. Allow use of 30° for $\frac{\pi}{6}$. **Dependent on the previous method mark.**	**d**M1
	$8 - 3\sqrt{3}$	Allow $8 - \sqrt{27}$. **Depends on not having lost any of the previous marks.**	A1
	Way 3: Cartesian Form		
	$y = 3\sin 2t = 6\sin t\cos t$	Uses $\sin 2t = 2\sin t\cos t$	M1
	$x = 4\sin t \Rightarrow \sin t = \frac{x}{4} \Rightarrow y = 6\left(\frac{x}{4}\right)\sqrt{1-\left(\frac{x}{4}\right)^2}$ Uses $\cos t = \sqrt{1-\sin^2 t}$ to obtain *y* in terms of *x*		M1
	$= \frac{3}{2}\int x\left(1-\frac{x^2}{16}\right)^{\frac{1}{2}}\ (dx)$	Correct integral or equivalent e.g. $\int\left(\frac{9}{4}x^2 - \frac{9}{64}x^4\right)^{\frac{1}{2}}\ (dx)$	A1
	$\frac{3}{2}\int x\left(1-\frac{x^2}{16}\right)^{\frac{1}{2}}\ (dx) = k\left(1-\frac{x^2}{16}\right)^{\frac{3}{2}}$	Correct form for the integration.	M1
	$\frac{3}{2}\int x\left(1-\frac{x^2}{16}\right)^{\frac{1}{2}}\ (dx) = -8\left(1-\frac{x^2}{16}\right)^{\frac{3}{2}}$	Correct integration	A1
	$\left[-8\left(1-\frac{x^2}{16}\right)^{\frac{3}{2}}\right]_0^2 = -8\left(1-\frac{2^2}{16}\right)^{\frac{3}{2}} + 8(1)$	Correct use of limits for their integrated function. Must see use of both limits. **Dependent on the previous method marks.**	**d**M1
	$8 - 3\sqrt{3}$	Allow $8 - \sqrt{27}$. **Depends on not having lost any of the previous marks.**	A1

Question Number	Scheme		Marks
9	$\lambda = -2 \rightarrow (6, -1, -5)$	Correct coordinates, values or vector seen or used or implied. These values are sometimes seen embedded with the work as e.g. 2 – 2(–2), 1 – 2, and 3 + 4(–2).	B1
	$3 - 2\mu = "-5" \Rightarrow \mu = ...$ $10 + \mu a = "6" \Rightarrow a = ...$	Uses the *z* component of l_2 to find μ and then uses the *x* component to find *a*	M1
	$a = -1$	Correct value for *a*	A1
	$\Rightarrow -2a + b - 8 = 0 \Rightarrow b = (6)$ A full method of finding "*b*". This involves using the fact that $\begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix} . \begin{pmatrix} a \\ b \\ -2 \end{pmatrix} = 0$ and using their value of *a*.		M1
	$"c + \mu b" = "-1" \Rightarrow c + 24 = -1 \Rightarrow c = (-25)$ A full method of finding "*c*" using the **j** coordinate. **Dependent on both previous method marks**		**dd**M1
	$b = 6,\ c = -25$	Correct values	A1
			(6)
			[6 marks]

Question Number	Scheme		Marks
10(a)	$\underline{3y^2\frac{dy}{dx}}+\underline{\underline{4x^2\frac{dy}{dx}}}+8xy-2=0$	$y^3 \to Ay^2\frac{dy}{dx}$	<u>M1</u>
		$4x^2y \to px^2\frac{dy}{dx}+qxy$	$\underline{\underline{\text{M1}}}$
		$3y^2\frac{dy}{dx}+4x^2\frac{dy}{dx}+8xy-2=0$ The "= 0" may be implied	A1
	$\left(3y^2+4x^2\right)\frac{dy}{dx}=2-8xy \Rightarrow \frac{dy}{dx}=\ldots$	Collects terms in $\frac{dy}{dx}$ (must be two and from the correct terms) and makes $\frac{dy}{dx}$ the subject of the formula	M1
	$\frac{dy}{dx}=\frac{2-8xy}{3y^2+4x^2}$	Correct expression or correct equivalent	A1
			(5)

Ignore any spurious "$\frac{dy}{dx}=$" for the first 3 marks

Allow full recovery in (b) if they have an incorrect denominator in (a)

(b)	$2-8xy=0 \Rightarrow y=\frac{1}{4x}$ or $2-8xy=0 \Rightarrow x=\frac{1}{4y}$	Sets the numerator of their $\frac{dy}{dx}=0$ and proceeds to $y = f(x)$ or $x = f(y)$	M1
	Note that starting with $2-8xy=4x^2+3y^2$ generally will score no marks in (b) **Note** that working with $4x^2+3y^2=0$ generally will score no marks in (b) and can be ignored if seen alongside work dealing with $2-8xy=0$ unless it yields extra spurious values – in which case the final mark can be withheld		
	$x=\frac{1}{4y} \Rightarrow y^3+4\left(\frac{1}{4y}\right)^2 y-2\left(\frac{1}{4y}\right)=0$ or $y=\frac{1}{4x} \Rightarrow \left(\frac{1}{4x}\right)^3+4x^2\left(\frac{1}{4x}\right)-2x=0$ Substitutes their x in terms of y or their y in terms of x into the equation for C **Dependent on the previous mark**		**d**M1
	$4y^4=1$ or $64x^4=1$	Correct simplified equation (allow equivalent forms e.g. $y^4=\frac{1}{4}$, $x^{-4}=64$)	A1
	$y=\frac{1}{\sqrt[4]{4}} \Rightarrow x=\ldots$ or $x=\frac{1}{\sqrt[4]{64}} \Rightarrow y=\ldots$ Substitutes at least one of their values of x or y to find a value for the other variable **or** Starts again and repeats the above process for the other variable leading to non-zero real values **Dependent on both previous method marks**		**dd**M1
	$x=\pm\frac{\sqrt{2}}{4},\ y=\pm\frac{\sqrt{2}}{2}$ **The points do not have to be explicitly given as coordinates so just look for values but if any extra points/coordinates are given the final mark should be withheld**		
	Two correct values for x or y or a correct pair (likely to be $x=\frac{\sqrt{2}}{4},\ y=\frac{\sqrt{2}}{2}$) For x allow e.g. : $\pm\frac{1}{\sqrt[4]{64}},\pm\frac{1}{2\sqrt{2}},\pm\frac{\sqrt{2}}{4},\pm 64^{-\frac{1}{4}}$ awrt ± 0.354 For y allow e.g. : $\pm\frac{1}{\sqrt[4]{4}},\pm\sqrt{\frac{1}{2}},\pm\frac{1}{\sqrt{2}},\pm\frac{\sqrt{2}}{2},\pm 4^{-\frac{1}{4}}$ awrt ± 0.707		A1
	All 4 values correct and exact and simplified For x allow e.g. : $\pm\frac{1}{2\sqrt{2}},\ \pm\frac{\sqrt{2}}{4}$ For y allow e.g. : $\pm\frac{1}{\sqrt{2}},\pm\frac{\sqrt{2}}{2},\pm\sqrt{\frac{1}{2}}$ Pairing is required but may be implied by e.g. $x=\pm\frac{1}{2\sqrt{2}}, y=\pm\frac{1}{\sqrt{2}}$. If after seeing correct values, the pairings are incorrect the final mark should be withheld.		A1
			(6)
			[11]

Note that it is possible to score M1dM1A1ddM0A1A0 if 2 values of one of x or y are found

Question Number	Scheme		Marks
11(a) Way 1	$\equiv \dfrac{\cos 3\theta \cos\theta + \sin 3\theta \sin\theta}{2\sin\theta\cos\theta}$	For forming a single fraction with a common denominator of $k\sin\theta\cos\theta$ with f(θ) + g(θ) in the numerator with at least one of f(θ) or g(θ) correct for their denominator	M1
	$\equiv \dfrac{\cos(3\theta-\theta)}{\ldots}$ or $\dfrac{\cos 2\theta}{\ldots}$ **or** $\equiv \dfrac{\ldots}{\sin 2\theta}$	For attempting to use a compound angle formula on the numerator **or** for attempting to $k\sin\theta\cos\theta = A\sin 2\theta$ on the denominator	M1
	$\equiv \dfrac{\cos(3\theta-\theta)}{\sin 2\theta}$ or $\dfrac{\cos 2\theta}{\sin 2\theta}$ For attempting to use a compound angle formula on the numerator **and** attempting to use $k\sin\theta\cos\theta = A\sin 2\theta$ on the denominator to reach $\dfrac{A\cos 2\theta}{B\sin 2\theta}$		M1
	$\equiv \cot 2\theta$ *	Proceeds to correct answer with all intermediate work and no errors or omissions. An error includes missing and/or inconsistent variables.	A1*
			(4)
(a) Way 2	$\equiv \dfrac{\cos 2\theta\cos\theta - \sin 2\theta \sin\theta}{2\sin\theta} + \dfrac{\sin 2\theta \cos\theta + \cos 2\theta \sin\theta}{2\cos\theta}$ For attempting to use a compound angle formula on the numerators with at least one correct		M1
	$\equiv \cos 2\theta\left(\dfrac{\sin^2\theta + \cos^2\theta}{2\sin\theta\cos\theta}\right)$	For forming a single fraction with a common denominator of $k\sin\theta\cos\theta$, factoring out $\cos 2\theta$ and simplifying the numerator.	M1
	$\equiv \cos 2\theta \times \dfrac{1}{\sin 2\theta}$ For attempting to use $k\sin\theta\cos\theta = A\sin 2\theta$ on the denominator **and** $\sin^2\theta + \cos^2\theta = 1$ on the numerator to reach $\dfrac{A\cos 2\theta}{B\sin 2\theta}$		M1
	Note that $\cos 2\theta \times \dfrac{1}{\sin 2\theta}$ can also be reached from $\cos 2\theta\left(\dfrac{\sin^2\theta + \cos^2\theta}{2\sin\theta\cos\theta}\right)$: $\cos 2\theta\left(\dfrac{\sin^2\theta+\cos^2\theta}{2\sin\theta\cos\theta}\right) = \dfrac{1}{2}\cos 2\theta(\tan\theta + \cot\theta) = \dfrac{1}{2}\cos 2\theta\left(\dfrac{1+\tan^2\theta}{\tan\theta}\right)$ $= \dfrac{1}{2}\cos 2\theta\left(\dfrac{\sec^2\theta}{\tan\theta}\right) = \dfrac{\cos 2\theta}{2\sin\theta\cos\theta} = \dfrac{\cos 2\theta}{\sin 2\theta}$ Award the third method mark for using correct trigonometry to reach $\dfrac{A\cos 2\theta}{B\sin 2\theta}$		
	$\equiv \cot 2\theta$ *	Proceeds to correct answer with all intermediate work and no errors or omissions. An error includes missing and/or inconsistent variables.	A1*

(a) Way 3	$\equiv \dfrac{\cos 3\theta\cos\theta + \sin 3\theta\sin\theta}{2\sin\theta\cos\theta}$	For forming a single fraction with a common denominator of $k\sin\theta\cos\theta$ with f(θ) + g(θ) in the numerator with at least one of f(θ) or g(θ) correct for their denominator	M1
	$\equiv \dfrac{\frac{1}{2}\cos 2\theta - \frac{1}{2}\cos 4\theta + \frac{1}{2}\cos 4\theta + \frac{1}{2}\cos 2\theta}{2\sin\theta\cos\theta}$ Applies the factor formulae to obtain $k(\cos 2\theta + \cos 4\theta)$ for $\cos 3\theta\cos\theta$ and $k(\cos 4\theta - \cos 2\theta)$ for $\sin 3\theta\sin\theta$		M1
	$\equiv \dfrac{\cos 2\theta}{\sin 2\theta}$ For attempting to use $k\sin\theta\cos\theta = A\sin 2\theta$ on the denominator **and** simplifies the numerator to reach $\dfrac{A\cos 2\theta}{B\sin 2\theta}$		M1
	$\equiv \cot 2\theta$ *	Proceeds to correct answer with all intermediate work and no errors or omissions. An error includes missing and/or inconsistent variables.	A1*
(a) Way 4	$\equiv \dfrac{\cos 3\theta\cos\theta + \sin 3\theta\sin\theta}{2\sin\theta\cos\theta}$	For forming a single fraction with a common denominator of $k\sin\theta\cos\theta$ with f(θ) + g(θ) in the numerator with at least one of f(θ) or g(θ) correct for their denominator	M1
	$\equiv \dfrac{\cos\theta\left(4\cos^3\theta - 3\cos\theta\right) + \sin\theta\left(3\sin\theta - 4\sin^3\theta\right)}{2\sin\theta\cos\theta}$ Applies the formulae for $\cos 3\theta$ and $\sin 3\theta$ to the numerator of their fraction. **If these formulae are quoted they must be correct otherwise a complete method must be seen to establish both of them**		M1
	$\equiv \dfrac{4\left(\cos^4\theta - \sin^4\theta\right) + 3\left(\sin^2\theta - \cos^2\theta\right)}{\sin 2\theta} = \dfrac{4\cos 2\theta - 3\cos 2\theta}{\sin 2\theta}$ Collects terms and applies $\cos^2\theta - \sin^2\theta = \cos 2\theta$ in the numerator **and** $k\sin\theta\cos\theta = A\sin 2\theta$ in the denominator to reach $\dfrac{A\cos 2\theta}{B\sin 2\theta}$		M1
	$\equiv \cot 2\theta$ *	Proceeds to correct answer with all intermediate work and no errors or omissions. An error includes missing and/or inconsistent variables.	A1*

(b)	$\cot 2x = 5\cos 2x \Rightarrow \sin 2x = \frac{1}{5}$ $(\cos 2x = 0)$	Uses $\cot 2x = \frac{\cos 2x}{\sin 2x}$ and proceeds to $\sin 2x = k$ $(-1 < k < 1)$	M1
	$\Rightarrow x = \frac{1}{2}\arcsin\frac{1}{5}$	Correct order of operations to find one value of x from $\sin 2x = k$ **Dependent on the previous mark**	**d**M1
	$\Rightarrow x = 0.101, 1.470, \frac{\pi}{4}\ (\text{or } 0.785)$ A1: Any 2 values which round to those shown. Allow $\frac{\pi}{4}$ or awrt 0.785 and allow 1.47 for 1.470 but not awrt 1.47 A1: All values which round to those shown. Allow $\frac{\pi}{4}$ or awrt 0.785 and allow 1.47 for 1.470 but not awrt 1.47 Ignore extra answers outside the range but withhold the final mark for extra answers in the range. **Answers in degrees lose both marks but ignore degrees symbols if present if the answers are otherwise correct**		A1A1
			(4)
			[8 marks]

Note that it is possible to answer Q12 using integration by parts (either way round) BUT it is very demanding and candidates are unlikely to get very far and will gain no marks.
If they reach $Ax + B\ln x + C\ln(x-4)$, $A, B, C \neq 0$ send to review.

Question Number	Scheme		Marks
12	$\dfrac{A}{x}+\dfrac{B}{x-4}=-\dfrac{2}{x}+\dfrac{14}{x-4}$	For an attempt to find partial fractions of the form $\dfrac{A}{x}+\dfrac{B}{x-4}$ where A and B are numeric and non-zero	M1
		Correct fractions $-\dfrac{2}{x}+\dfrac{14}{x-4}$	A1
	$\dfrac{3x^2+8}{x^2-4x}=3+\mathrm{f}(x)$ Where $\mathrm{f}(x)=\dfrac{A}{x}+\dfrac{B}{x-4}$ with numeric *A* and *B* or the letters "*A*" and "*B*" **or** Where $\mathrm{f}(x)=\dfrac{Cx+D}{x^2-4x}$ with numeric *C* and *D* with *C*, *D* not both zero		B1
	This mark is for integrating at least 2 terms of the form $\dfrac{\alpha}{x\pm k}$ to obtain $\beta\ln(x\pm k)$ where *k* may be zero Allow e.g. ln($x \pm k$), ln($k \pm x$), $\ln\lvert x\pm k\rvert$, also allow ln $x \pm k$ for this mark		M1
	For $\int 3-\dfrac{2}{x}+\dfrac{14}{x-4}\,\mathrm{d}x \to 3x-2\ln\lvert x\rvert+14\ln\lvert x-4\rvert$ following through on their coefficients requires modulus signs and/or brackets around the $x - 4$ unless they are implied by later work. E.g. allow $3x-2\ln x+14\ln(x-4)$		A1ft
	$=9-2\ln 3-3-14\ln 3=...$ Evidence of the use of both limits 3 and 1 and subtracts the right way round and reaches an expression of the form $P + Q\ln R$, where *P*, *Q* and *R* are rational and non-zero and $R > 0$ **Dependent on the previous method mark**		**d**M1
	$=6-16\ln 3$ Accept equivalents e.g. $6-8\ln 9, 6+16\ln\left(\dfrac{1}{3}\right), 6-\ln 3^{16}, 6+\ln\dfrac{1}{43046721}, 6-\ln 43046721$ $6+\ln\dfrac{1}{9\times 3^{14}}, 6-\ln\left(9\times 3^{14}\right)$ etc.		A1
			(7)
			[7 marks]

Special Case:

Some students know to use PF but fail to see it is an improper fraction and the solution will look similar to this:

$$\frac{3x^2+8}{x^2-4x}=\frac{14}{x-4}-\frac{2}{x}$$

$$\int_1^3 \frac{3x^2+8}{x^2-4x}\,\mathrm{d}x=\int_1^3 \frac{14}{x-4}-\frac{2}{x}\,\mathrm{d}x=\left[14\ln\lvert x-4\rvert-2\ln\lvert x\rvert\right]_{(x=1)}^{(x=3)}$$

$$=14\ln 1-2\ln 3-(14\ln 3-2\ln 1)=-16\ln 3$$

These students can potentially score **M1 A1 B0 M1 A0 dM0 A0** for 3 out of 7

Question Number	Scheme		Marks
13(a)	$\frac{dy}{dt}=\frac{2(1-t)-2t\times -1}{(1-t)^2}$ or $\frac{dy}{dt}=2(1-t)^{-1}+2t(1-t)^{-2}$ M1: If the **quotient** rule is not quoted and u, v, u', v' **are not** stated they must obtain $\frac{A(1-t)\pm Bt}{(1-t)^2}\quad A,B>0$ If the **quotient** rule is not quoted and u, v, u', v' **are** stated they must be correctly positioned in the quotient rule If the **quotient** rule is quoted it must be correct **Or** M1: If the **product** rule is not quoted and u, v, u', v' **are not** stated they must obtain $A(1-t)^{-1}\pm Bt(1-t)^{-2}\quad A,B>0$ If the **product** rule is not quoted and u, v, u', v' **are** stated they must be correctly positioned in the product rule If the **product** rule is quoted it must be correct A1: Correct $\frac{dy}{dt}$ in any form.		M1 A1
	$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{\frac{2}{(1-t)^2}}{2t+3}$	Uses $\frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}$	M1
	$=\frac{2}{(2t+3)(1-t)^2}$	Correct expression. Allow the $(1-t)^2$ to expanded as long as it is collected and allow the whole of the denominator to be expanded as long as it is collected but apply isw where possible.	A1
			(4)
(b)	$\left(\frac{dy}{dx}\right)_{t=2}=\frac{2}{(2(2)+3)(1-2)^2}\left(=\frac{2}{7}\right)$	For substituting $t=2$ in their $\frac{dy}{dx}$	M1
	$t=2\Rightarrow x=10,\ y=-4$	Correct coordinates for P	B1
	$y+4=\frac{2}{7}(x-10)$ or $-4=\frac{2}{7}(10)+c\Rightarrow c=...$	Correct method for the equation of the tangent using their P.	M1
	$\Rightarrow 2x-7y-48=0$	Or any integer multiple of this. If the $\frac{2}{7}$ is obtained fortuitously then this mark should be withheld.	A1cso
			(4)

If $2x-7y-48=0$ is obtained fortuitously in (b) all the marks are available in (c) apart from the A1cso

(c) Way 1	$x=t^2+3t,\ y=\frac{2t}{1-t},\Rightarrow 2x-7y-48=0\Rightarrow 2\left(t^2+3t\right)-7\left(\frac{2t}{1-t}\right)-48=0$ Uses the given parametric coordinates and substitutes into their tangent to form an equation in t		M1
	$2t^3+4t^2-40t+48=0$	Correct equation	A1
	If they have a correct cubic equation and the root $t=-6$ is seen, this method can be implied. **If they do not have the correct equation, to score this mark they must have obtained a cubic equation that has a constant term and they need to attempt to factorise using $(t\pm 2)$ or $(t\pm 2)^2$ as a factor.** Look for $(t\pm 2)(at^2+...)$ or $(t\pm 2)^2(at+...)$ or may use long division so look for the corresponding expressions for the quotient e.g. $at^2+\ldots$ or $at+\ldots$ **Depends on the first method mark.**		**d**M1
	$t=-6$	Correct value for t **that has come from a correct cubic**.	A1
	$Q=\left(18,-\frac{12}{7}\right)$	Correct coordinates. Allow $x=18,\ y=-\frac{12}{7}$ Ignore any reference to any other points e.g. (10, –4)	A1cso
			(5)
(c) Way 2	$y=\frac{2t}{1-t}\Rightarrow t=\frac{y}{y+2}\Rightarrow 2\left(\left(\frac{y}{y+2}\right)^2+3\left(\frac{y}{y+2}\right)\right)-7y-48=0$ Finds t in terms of y and substitutes into their tangent equation to form an equation in y. When eliminating t using y, the algebra must be correct so allow sign errors only for making t the subject from y.		M1
	$7y^3+68y^2+208y+192=0$	Correct equation	A1
	If they have a correct cubic equation and the root $y=-12/7$ is seen, this method can be implied. **If they do not have the correct equation, to score this mark they must have obtained a cubic equation that has a constant term and they need to attempt to factorise using $(y\pm 4)$ or $(y\pm 4)^2$ as a factor.** Look for $(y\pm"-4")(ay^2+...)$ or $(y\pm"-4")^2(ay+...)$ or may use long division so look for the corresponding expressions for the quotient e.g. $ay^2+\ldots$ or $ay+\ldots$ **Depends on the first method mark.**		dM1
	$y=-\frac{12}{7}$	Correct value for y **that has come from a correct cubic**.	A1
	$x=18$	Correct value for x	A1cso
	Ignore any reference to any other points e.g. (10, –4)		
			[13 marks]

Question Number	Scheme		Marks
14(a)	360	Cao. No need for $N =$ … just look for the correct value	B1
			(1)
(b)	900	Cao. No need for $N =$ … just look for the correct value. Allow e.g. $N < 900$	B1
			(1)
(c)	$780 = \dfrac{1800}{2+3\mathrm{e}^{-0.2t}} \Rightarrow 2340\mathrm{e}^{-0.2t} = 240$	Substitutes $N = 780$ and proceeds to $A\mathrm{e}^{\pm 0.2t} = B$, where A and B are both positive or both negative	M1
	$2340\mathrm{e}^{-0.2t} = 240$	Correct equation oe e.g. $\mathrm{e}^{-0.2t} = \dfrac{4}{39}$, $3\mathrm{e}^{-0.2t} = \dfrac{4}{13}$, $\mathrm{e}^{0.2t} = \dfrac{39}{4}$	A1
	$2340\mathrm{e}^{-0.2t} = 240 \Rightarrow \mathrm{e}^{-0.2t} = \dfrac{4}{39} \Rightarrow -0.2t = \ln\left(\dfrac{4}{39}\right) \Rightarrow t = ...$ **or** $2340\mathrm{e}^{-0.2t} = 240 \Rightarrow \ln 2340\mathrm{e}^{-0.2t} = \ln 240 \Rightarrow \ln \mathrm{e}^{-0.2t} = \ln 240 - \ln 2340$ $\mathrm{e}^{-0.2t} = \dfrac{4}{39} \Rightarrow -0.2t = \ln\left(\dfrac{4}{39}\right) \Rightarrow t = ...$ This mark is for **fully correct processing** from $A\mathrm{e}^{\pm 0.2t} = B$ to obtain a value for t **Dependent on the first method mark**		**d**M1
	$(t =)$ 11.4	For awrt 11.4	A1
			(4)

(d)(i) Way 1	Quotient: $\frac{dN}{dt}=\frac{\left(2+3e^{-0.2t}\right)\times 0-1800\times -0.6\times e^{-0.2t}}{\left(2+3e^{-0.2t}\right)^2}$ Chain: $\frac{dN}{dt}=-1800\times -0.6\times e^{-0.2t}\left(2+3e^{-0.2t}\right)^{-2}$ M1: For obtaining a derivative of the form $\frac{Ae^{-0.2t}}{\left(2+3e^{-0.2t}\right)^2}$ A1: Correct derivative in any form which may be unsimplified as above. Often seen as $\frac{1080e^{-0.2t}}{\left(2+3e^{-0.2t}\right)^2}$	M1 A1
	$\Rightarrow \frac{dN}{dt}=\frac{1800\times 0.6\times \frac{1}{3}\left(\frac{1800}{N}-2\right)}{\left(\frac{1800}{N}\right)^2}$ A full attempt to get $\frac{dN}{dt}$ in terms of N. Both $e^{-0.2t}$ and $\left(2+3e^{-0.2t}\right)^2$ must be replaced by a function of N. **Dependent on the first method mark**	**d**M1
	$\Rightarrow \frac{dN}{dt}=\frac{N(900-N)}{4500}\quad \therefore A=4500$ | $\frac{dN}{dt}=\frac{N(900-N)}{4500}$	A1
(d)(i) Way 2	$N=\frac{1800}{2+3e^{-0.2t}}\Rightarrow N\left(2+3e^{-0.2t}\right)=1800\Rightarrow\left(2+3e^{-0.2t}\right)\frac{dN}{dt}+N\left(-0.6e^{-0.2t}\right)=0$ M1: $\left(2+3e^{-0.2t}\right)\frac{dN}{dt}+Ae^{-0.2t}=0$ A1: Correct equation **or** $N=\frac{1800}{2+3e^{-0.2t}}\Rightarrow 2N+3Ne^{-0.2t}=1800\Rightarrow 2\frac{dN}{dt}+3e^{-0.2t}\frac{dN}{dt}+N\left(-0.6e^{-0.2t}\right)=0$ M1: $A\frac{dN}{dt}+Be^{-0.2t}\frac{dN}{dt}+CNe^{-0.2t}=0$ A1: Correct equation	M1A1
	$\frac{dN}{dt}=\frac{0.6Ne^{-0.2t}}{2+3e^{-0.2t}}=\frac{0.6N\left(\frac{1800}{3N}-\frac{2}{3}\right)}{\frac{1800}{N}}$ Makes $\frac{dN}{dt}$ the subject and a full attempt to get $\frac{dN}{dt}$ in terms of N. Both $e^{-0.2t}$ and $2+3e^{-0.2t}$ must be replaced by a function of N. **Dependent on the first method mark**	**d**M1
	$\Rightarrow \frac{dN}{dt}=\frac{N(900-N)}{4500}\quad \therefore A=4500$ | $\frac{dN}{dt}=\frac{N(900-N)}{4500}$	A1

(d)(i) Way 3	$N = \dfrac{1800}{2+3\mathrm{e}^{-0.2t}} \Rightarrow 2N + 3N\mathrm{e}^{-0.2t} = 1800 \Rightarrow \mathrm{e}^{-0.2t} = \dfrac{1800-2N}{3N}$ $\Rightarrow -0.2t = \ln\left(\dfrac{1800-2N}{3N}\right) \Rightarrow \dfrac{\mathrm{d}t}{\mathrm{d}N} = -5\times\left(\dfrac{3N}{1800-2N}\right)\times -600N^{-2}$ M1: For an attempt to make t or $-0.2t$ the subject and then applies the chain rule to obtain $\dfrac{\mathrm{d}t}{\mathrm{d}N}$ A1: Correct derivative in any form	M1 A1
	$\Rightarrow \dfrac{\mathrm{d}N}{\mathrm{d}t} = \dfrac{(1800-2N)N^2}{9000}$ A full attempt to get $\dfrac{\mathrm{d}N}{\mathrm{d}t}$ in terms of N. **Dependent on the first method mark**	**d**M1
	$\Rightarrow \dfrac{\mathrm{d}N}{\mathrm{d}t} = \dfrac{N(900-N)}{4500} \quad \therefore A = 4500$ $\dfrac{\mathrm{d}N}{\mathrm{d}t} = \dfrac{N(900-N)}{4500}$	A1
(ii)	$N = 450$ Cao	B1
		(5)
		[11marks]

Question Number	Scheme		Marks
15(a)	$\frac{8000}{56+9+0}=\frac{8000}{65}=\frac{1600}{13}$	Allow any equivalent fraction or awrt 123m	B1
			(1)
(b)	$9\cos t+40\sin t=R\cos(t-\alpha)$		
	$R=\sqrt{9^2+40^2}=41$	41 only	B1
	$\alpha=\arctan\left(\pm\frac{40}{9}\right)=...$ or $\alpha=\arctan\left(\pm\frac{9}{40}\right)=...$ or $\alpha=\arcsin\left(\pm\frac{40}{"41"}\right)=...$ or $\alpha=\arccos\left(\pm\frac{9}{"41"}\right)=...$		M1
	$\alpha=77.3$	Awrt 77.3	A1
			(3)
(c)(i)	$\frac{8000}{56+'R'}=...$ m	Attempts $\frac{8000}{56+'R'}$	M1
	$=\frac{8000}{97}$	$\frac{8000}{97}$ or awrt 82.5	A1
(ii)	$t=77.3$	Awrt 77.3 or follow through their α (ignore what they do in (c)(i))	B1ft
			(3)
(d)	$150=\frac{8000}{56+41\cos(t-77.3)}\Rightarrow\cos(t-77.3)=-0.065$ Uses their part (b) with H = 150 and reaches $\cos(t\pm77.3)=k$ with $-1<k<0$		M1
	$\cos(t\pm"77.3")=-\frac{8}{123}$ or awrt -0.065 (Follow through their 77.3)		A1ft
	$\cos(t\pm77.3)=-\frac{8}{123}\Rightarrow t\pm77.3=\arccos\left(-\frac{8}{123}\right)\Rightarrow t=...$ Takes arccos and then ±"77.3" and uses the **<u>obtuse</u>** angle leading to a value for t **Dependent on the first M so requires** $-1<k<0$		**d**M1
	$(t=)171$	Awrt 171 **and no other values**	A1
			(4)
			[11 marks]

Note that the use of radians for an otherwise correct solution would normally lose the A mark in (b) and the final A mark in (d). (Values are (a) 1.349 and (d) 2.98)

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